Formation of Nonlinear Stationary Structures in Ionospheric Plasma

Gobinda Manna, Suman Dey, Jyotirmoy Goswami, Swarniv Chandra, Jit Sarkar, and Amrita Gupta

***Abstract*— Solar radiation, along with cosmic radiation, ionizes the Earth's atmosphere and creates a dense layer known as the ionosphere. By considering weakly relativistic degenerate plasma in the planetary ionosphere, we have studied the formation and nature of the solitary structure, electrostatic double layers (DLs), and so on. As we have considered weak relativistic degeneracy, only electrons get accelerated and are the key to stationary structures. We have implemented Sagdeev's pseudopotential method, standard Gardner equation, and accordingly identified regimes where the solitary formation and DLs may be observed. We have studied the parametric influences on solitons and DLs. Furthermore, we extended our investigation to the oscillatory Rossby solitons in the ionospheric plasma. The results may help interpret many high-energy atmospheric observations in the ionospheric plasma.**

***Index Terms*— Double layers (DLs), Gardner soliton, ionosphere plasma, Korteweg–de Vries (KdV), modified Korteweg–de Vries (mKdV), oscillatory Rossby wave (RW), pseudopotential, relativistic degeneracy, solitons.**

I. Introduction

SOLAR radiation, along with cosmic radiation, ionizes the Earth's atmosphere and creates a dense layer known as the ionosphere. It is located between 80 and 950 km (50 and 590 miles) above the Earth. It is composed of three main parts, which are the D, E, and F zones. The widespread existence of naturally excited electrostatic wave has been detected by wave experiment onboard Freja spacecraft [1]. Fast Auroral Snapshot (FAST) observations gave a transparent indication of the existence of the large-amplitude solitary structures in the Earth's ionosphere. In Titan's ionosphere, various nonlinear stationary structures such as a solitary wave (SW), explosive, shock-like, and periodic waves are studied by Ahmed *et al.* [2]. In the Venusian ionosphere, small but finite amplitude SW, small-amplitude double layers (DLs), and Gardner soliton have been derived by Moslem *et al.* [3]. In many situations where the particle density is high, there arises a type of degeneracy known as relativistic degeneracy. Initial researchers like Eddington [4] and Chandrasekhar [5] worked on such dense degenerate matter. Among them, Chandrasekhar came with an expansion for relativistic degeneracy pressure. In the presence of relativistic degeneracy pressure, instabilities have played a major role in many nonlinear phenomena in plasma [6], [7]. Various dynamical properties of shock and solitary structures have been studied in degenerate Plasma [8]. Electron acoustic (EA) waves in ultrarelativistic dense degenerate plasmas have been observed by Rahman *et al.* [9]. The existence of EA solitons in a two-electron temperature plasma ("cold" and "hot" electrons) has been studied by Ghosh *et al.* [10], Lakhina and Singh [11], Devanandhan *et al.* [12]–[14], and Singh *et al.* [15]. EA and shocks on solitary structures have been scrutinized in the dense inner magnetosphere [16]. Interaction of laser beam with dense plasma creates envelop soliton where the amplitude is modulated [17]. Amplitude modulated solitons [18], [19] are evolved from DL structures through shock fronts and converge into EA solitary structures in finite temperature quantum plasma [20]. To investigate electrostatic acoustic solitary structures, Mukhopadhyay *et al.* [21] used a 1-D quantum hydrodynamic (QHD) model in two-component homogeneous plasma. Nonlinear EA waves have been analyzed by Bala *et al.* [22], Devanandhan *et al.* [23], Mbuli *et al.* [24], and Singh *et al.* [25] in an unmagnetized plasma system (containing cold and hot electrons) to explore the existence of large- and small-amplitude solitary and DLs using the Sagdeev pseudopotential approach. Lakhina *et al.* [26] reviewed the nonlinear fluid model in multicomponent plasma for DLs and EA wave. Ion acoustic SWs have been investigated by many authors using Sagdeev pseudopotential approach for two-component plasma [27], three-component plasma [28], and four-component plasma [29], [30].

Korteweg–de Vries (KdV) equation has been obtained by many authors for weakly relativistic plasma propagating in a collision-less plasma. Relativistic effect influences the amplitude, the phase velocity, and the width of an SW solution (Chandra *et al.* [31], Sarkar *et al.* [32]). Investigation of solitary profiles and shocks has been carried out in collision-less quantum plasma by Goswami *et al.* [33]. Envelope soliton of waves has been formed in the space [34]. Sahoo *et al.* [35], Roychowdhury *et al.* [36], Ballav *et al.* [37], Thakur *et al.* [38], and Goswami *et al.* [39] studied the formation of stationary structure in dusty plasma. Many researchers [40], [41] have also observed small-amplitude ion acoustic SWs. The KdV equation in the presence of

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an external force has been derived by Chandra *et al.* [42]. Further extension of nonlinearity to higher order put forward to modified Korteweg–de Vries (mKdV) equation in weakly relativistic degenerate plasma containing hot and cold electrons and finite ions [43]. For EA wave in a four-component plasma, small-amplitude mKdV solitary structures are investigated by El-Taibany *et al.* [44]. Ghosh *et al.* [45] inspected that EA SWs propagate in semiclassical plasma and provoke rogue waves where plasma contains two different temperatures' electrons and cold ions. Dey *et al.* [46] and Samanta *et al.* [47] studied the formation of rogue wave in ion acoustic mode for the highly energetic case and degenerate plasma. The KdV equation with small nonlinearity is described as the standard Gardner (SG) equation. However, in the Coastal Ocean Probe Experiment in 1995, strong nonlinearity is also experienced [48]. Nonlinear effects generate the second harmonic in high-density quantum plasma [49]. Propagation of different harmonics (higher order) depends on the phase velocities [50]. Gardner solitons have been studied by Deeba *et al.* [51] in a four-component dusty plasma, whereas Allehiany *et al.* [52] have investigated solitary and shock-like wave solutions for the Gardner equation with two-temperature ions and isothermal electron in dusty plasma.

The double-layer structures containing two parallel layers of opposite electrical charges in the space or in the laboratory-produced plasma have been widely studied [53]. They are thin sheet-like charge distributions, not necessarily planar which produce localized excursions of an electric potential. It results in a strong electric field between them but very weak yet extensive on the outside, which quickly stabilizes the surrounding potential of the environment. The wave propagation creates current (streams of the thickness of few Debye length), which gives rise to DLs [54]. When the potential drop within the layers is comparable to the remaining mass energy of electrons (512 KeV), the DLs are said to be relativistic. Such DLs are known as relativistic DLs [55]. In terms of strength, if the potential drop is often high compared with the thermal energy of particles (electrons), it is said to be strong DLs; else, it is considered weak. Experimental studies suggest that DLs occur under many plasma environments [56]. Where two species of charged particle, viz. electrons and ions, move in opposite directions with negligible collisional effects, thus creating spatially split two regions of the opposite polarity and a high potential drop. If a magnetic field is included, the charged particles flow in the magnetic field direction, provided the magnetic field is highly intense. Acceleration of electron is explored in magnetized quantum plasma [57]. The space plasma witnesses DLs with a potential drop of about 10^9–10^{11} V [55]. For instance, in the solar atmosphere, DLs are generated by thin current filaments that penetrate it (solar atmosphere) [55], [58]. Nonrelativistic DLs have been theorized by Langmuir [59] and Block [60]. In contrast, relativistic cases have not been studied quite well. The studies of Albert and Lindstrom [61], Wescott *et al.* [62], etc. reveal that a potential drop of about 10^2–10^4 may exist at an altitude of a few hundred to several thousand of kilometers from the Earth's surface (ionosphere). Relativistic and nonrelativistic classification is carried out by comparing the order of potential drop with the ion and electron remaining mass energies. If $\phi_{\mathrm{DL}} \ll (m_e c^2)/e$, such double layers are nonrelativistic, whereas if $\phi_{\mathrm{DL}} \gg (m_i c^2)/e$, it amounts to relativistic DLs. These simply imply that if the potential drop becomes much less than electron remaining mass energy, the DL fails to accelerate even the lightest electron. Recent studies by the Voyager space mission revealed a lot of information on DLs in space plasma [63]. Chen *et al.* [64] describe the first direct measurement of how energy from electro-magnetic waves in space is transferred to particles. The studies of Alfvn and Carlqvist [58] on the nature and formation of double-layer is highly informative. According to him, DLs are self-protecting entities in space plasma that encompass intense electric fields between them. From the SG equation, we can obtain DLs' solution. Both SW and DLs' solutions have been investigated from Gardner equation by Shuchy *et al.* [65], Hosen *et al.* [66], and Akhter *et al.* [67]. In magnetized quantum plasma, Atteya *et al.* [68] examined the ion acoustic Gardner solitons and DLs and revealed the existence of positive and negative DLs. For the first time, Russian scientist Dokuchaev [69] discussed the existence of planetary Rossby waves (RWs) in the ionosphere. Interaction of the spatially inhomogeneous geomagnetic field with the medium creates magnetized RWs. In the rotating dissipative ionosphere, Aburjania *et al.* [70] investigated the formation of planetary RWs, which are magnetized. Observations at Cariri Airglow Observatory (7.4 °S, 36.5 °W) in South America, roughly 1000 km to the east of *São Luís*, indicate the presence of planetary-scale oscillations and RWs in the ionosphere (Takahashi *et al.* [71]). A study by Tsintsadze *et al.* [72] revealed that the excitation of RWs in weakly ionized gas is influenced by the amplitude modulation of electromagnetic radiation.

In the present work, we have studied various stationary structures in ionospheric plasma by applying different techniques. First, to obtain large-amplitude SW and DLs, we use the Sagdeev pseudopotential approach. Then using the method of reductive perturbation, we obtain the KdV, mKdV, and SG equations around the critical regime to perceive small-amplitude Gardner soliton and DLs. We also studied the oscillatory Rossby SW and its dependence on plasma parameters for a three-component dense plasma with noninertial degenerate electrons, inertial cold nondegenerate electrons, and a substrate of heavy stationary ions that satisfy the quasi-neutrality conditions. As it is the ionospheric plasma, we have considered that the energy transfer to EA waves from Alfvén waves is due to the mode coupling and subsequent dissipation in the transition region as proposed by Shi *et al.* [73] and Barik *et al.* [74]. The article is designed in the following pattern. Starting from the dynamical equations in Section II, we obtain the derivation of Sagdeev pseudopotential, SW, and double-layer formation in Section III. Next, we study KdV SW in Section IV. In Section V at first, we derive modified KdV (mKdV) using slightly higher order stretching, then derive SG equation near-critical regime, and study the DLs' solution of SG soliton. Furthermore, we also study oscillatory Rossby soliton and its numerical solution in Section VI. Finally, we discuss all the results in Section VII and make a conclusion in Section VIII.

II. Basic Governing Equation

We studied the nonlinear propagation of an EA wave (EAW) in a plasma made up of inertia-less warm electrons [75] which are relativistically degenerate [76]. We have considered here weakly relativistic degeneracy [77] because we have comparatively warm electrons that are mobile and therefore supplying the necessary restoring force. The cold electrons, on the other hand, provide inertia so that the warm electron can execute oscillations about the mean positions. The ions exist as a stationary background to maintain a quasi-neutrality condition. Under these conditions, the set of nonlinear dynamic equations may be written as

$$\frac{\partial n_c}{\partial t} + \frac{\partial (n_c u_c)}{\partial x} = 0 \tag{1}$$

$$\frac{\partial n_h}{\partial t} + \frac{\partial (n_h u_h)}{\partial x} = 0 \tag{2}$$

$$\frac{\partial u_c}{\partial t} + u_c \frac{\partial u_c}{\partial x} = \alpha \frac{\partial \varphi}{\partial x} \tag{3}$$

$$n_h \frac{\partial \varphi}{\partial x} - \frac{3}{5}\beta \frac{\partial {n_h}^{\frac{5}{3}}}{\partial x} = 0 \tag{4}$$

$$\frac{\partial^2 \varphi}{\partial x^2} = n_h + \frac{1}{\alpha}(n_c) - \left(1 + \frac{1}{\alpha}\right) = -\rho \tag{5}$$

where $\alpha = (n_{h0})/(n_{c0})$ is the hot-to-cold electron equilibrium density ratio, β $(= K_0\gamma)$ is the coefficient for relativistic degeneracy pressure after normalization, and ρ is the normalized charge density, where $K_0 = (72\pi)^{-1/3}$ and $\gamma = \lambda_c n_{h0}^{1/3}$. Here, λ_c is the Compton wavelength. This relativistic degeneracy pressure for various dense stellar bodies has been considered by Goswami *et al.* [78] and Akbari-Moghanjoughi [79]. "EA envelope soliton in astrophysical plasma" was studied by Goswami and Sarkar [80]. The suffixes h, c, and i correspond to hot electrons, cold electrons, and ions, respectively.

III. Electrostatic Stationary Structure

Let us assume that all dependent variables are functions of a single independent variable to get a localized stationary solution. $\xi = x - Mt$. M is the Mach number defined by v/c_s, and v denotes the velocity of the nonlinear waveform propagating with the frame. The above equations can be expressed as

$$-M\frac{\partial n_c}{\partial \xi} + \frac{\partial (n_c u_c)}{\partial \xi} = 0 \tag{6}$$

$$-M\frac{d u_c}{d \xi} + u_c \frac{d u_c}{d \xi} = \alpha \frac{d\varphi}{d\xi} \tag{7}$$

$$\frac{d^2\phi}{d\xi^2} = \left(1+\frac{2\varphi}{3\beta}\right)^{\frac{3}{2}} + \frac{1}{\alpha}(n_c) - \left(1 + \frac{1}{\alpha}\right). \tag{8}$$

Now, integrating (3), one can express n_{h0} as $n_{h0} = (1 + (2\varphi)/(3\beta))^3/2$. Integrating (5) and (6) and putting the boundary condition (BC), we obtain

$$n_c = \frac{1}{\sqrt{1 + \frac{2\alpha\varphi}{M^2}}}. \tag{9}$$

Again

$$u_c = M\left(1 - \frac{1}{n_c}\right). \tag{10}$$

Substituting (8) into (7), multiplying $(d\varphi)/(d\xi)$ with the resultant equation, integrating once, and applying the BCs for localized solutions, the standard energy integral equation is as follows:

$$\frac{1}{2}\left(\frac{d\varphi}{d\xi}\right)^2 + V(\varphi) = 0 \tag{11}$$

where $V(\phi)$ is the Sagdeev pseudopotential. This can be expressed as

$$V(\varphi) = \frac{3\beta}{5}\left\{1 - \left(1 + \frac{2\varphi}{3\beta}\right)^{\frac{5}{2}}\right\} + \frac{M^2}{\alpha^2}\left\{1 - \sqrt{1 + \frac{2\alpha\varphi}{M^2}}\right\} + \left(1 + \frac{1}{\alpha}\right)\varphi. \tag{12}$$

The Sagdeev pseudopotential under various parametric variations may correspond to solitary structures, DLs, or shock waves. The following section outlines the limiting cases of amplitude expansion.

A. Small Amplitude Expansion

If the nonlinearity is low, then the quasiparticles in this pseudopotential make a single excursion, which is known as SW. To find out the solution of this type of solitons, let us expand the $V(\varphi)$ concerning and assuming the quantity φ is very small. We take the first two order of φ and get

$$\frac{d^2\varphi}{d\xi^2} = A_1\varphi - A_2\varphi^2 \tag{13}$$

where $A_1 = (1/\beta - 1/M^2)$ and $A_2 = -(1/(6\beta^2) + (3\alpha)/(2M^4))$. By a transformation $\varphi(\xi) = \mathrm{z}(w)$ with $w = \operatorname{sech}(\zeta'\xi)$, the above equation reduces to

$$\zeta'^2 w^2(1 - w^2)\frac{d^2 z}{dw^2} + \zeta' w(1 - 2w^2)\frac{dz}{dw} - A_1 z + A_2 z^2 = 0. \tag{14}$$

Since $w = 0$ is a regular singularity, we therefore attempt for a solution as follows [81]:

$$z(w) = \sum_{r=0}^{\infty} b_r w^r. \tag{15}$$

The series get truncated for $r = 3$ if $\zeta' = (A_1/4)^{1/2}$, $b_2 = (3A_1)/(2A_2)$, $b_0 = 0$, and $b_1 = 0$, and we get a solution in the form

$$\varphi = \left(\frac{3A_1}{2A_2}\right) \operatorname{sech}^2\left(\frac{\xi}{\delta}\right) \tag{16}$$

where $\delta = (4/A_1)^{1/2}$. Equation (16) is the expression for a KdV SW propagating along the ξ axis where $(3A_1)/(2A_2)$ is the amplitude and δ is the width of the corresponding soliton.

To study the formation and nature of solitary structures and DLs, we plot the Sagdeev pseudopotential $V(\varphi)$ versus φ with a parametric variation. These basic parameters are the hot-to-cold electron equilibrium density ratio (α), normalized

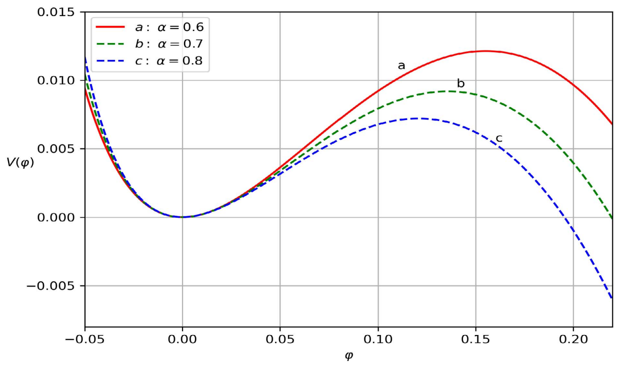

Fig. 1. Pseudopotential for different hot-to-cold electron density ratios (α) [82], where $\beta = 0.274$ and $M = 0.35$.

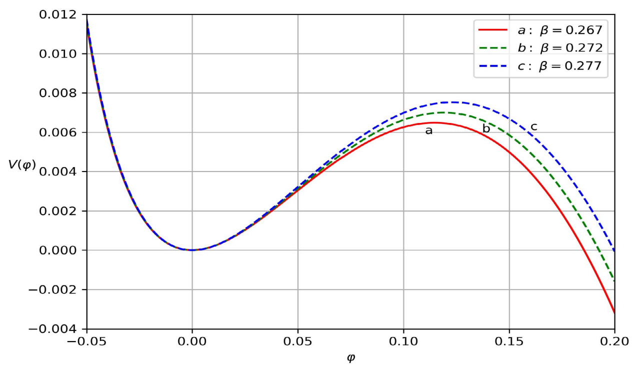

Fig. 2. Pseudopotential varies with different coefficients for the relativistic degeneracy pressure (β), where $\alpha = 0.8$ and $M = 0.35$.

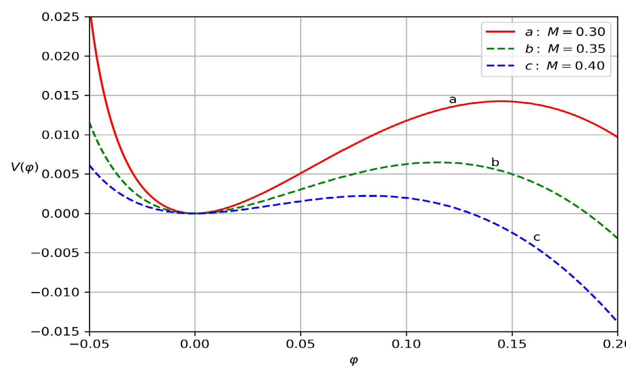

Fig. 3. Pseudopotential varies with different Mach numbers (M), where $\alpha = 0.8$ and $\beta = 0.274$.

relativistic factor (β), and Mach number (M). In Figs. 1–3, we plot the parametric dependence of Sagdeev pseudopotential with α, β, and Mach number (M). In Fig. 4, we show the solitary structures with varying hot-to-cold electron density ratios (α), keeping β and M constant. In Fig. 5, we show solitary structure varying with the coefficient for the relativistic degeneracy pressure (β) keeping α and M constant. We also show solitary structure for different Mach numbers keeping other parameters constant (Fig. 6). The contour plots in ξ–α, ξ–β, and ξ–M planes are shown in Figs. 7–9, respectively. We now go to a higher order expansion of the pseudopotential in (12).

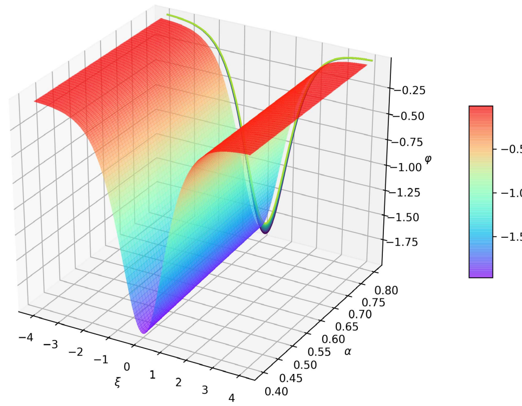

Fig. 4. SW structures for different ratios of hot to cold electron equation density (α) where $\beta = 0.272$ and $M = 1.4$.

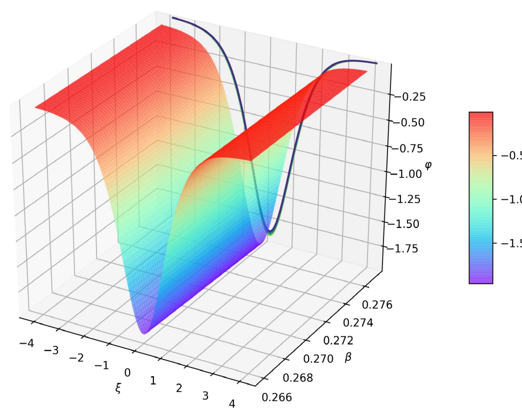

Fig. 5. Solitary structures varying with the coefficient for the relativistic degeneracy pressure (β), where $\alpha = 0.5$ and $M = 1.4$.

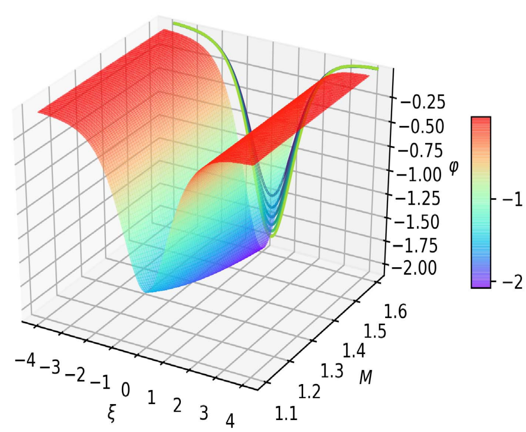

Fig. 6. SW structures for different Mach numbers (M) where $\alpha = 0.5$ and $\beta = 0.272$.

B. Slightly Higher Order Term Corrections

When there will be a higher nonlinearity than the previous one, then a localized potential jump will occur naturally, and to derive the solution of that let us now include φ^3 term in $(d^2\varphi)/(d\xi^2)$ and we can obtain

$$\frac{d^2\varphi}{d\xi^2} = A_1\varphi - A_2\varphi^2 + A_3\varphi^3 \tag{17}$$

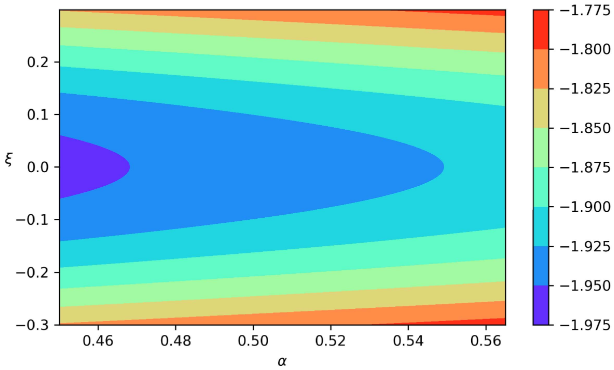

Fig. 7. Contour plot in the ξ–α plane for various α where $\beta = 0.272$ and $M = 1.4$.

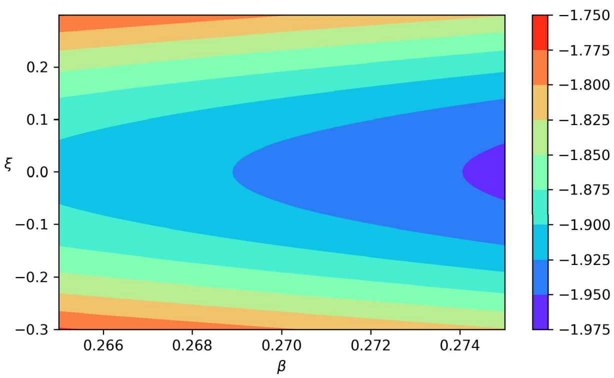

Fig. 8. Contour plot in the ξ–β plane for different coefficients for the relativistic degeneracy pressure (β), where $\alpha = 0.5$ and $M = 1.4$.

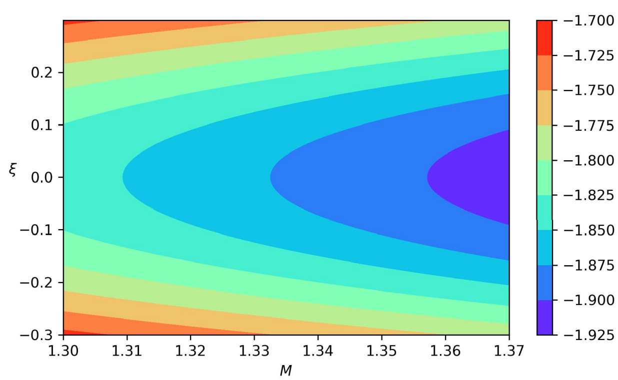

Fig. 9. Contour plot in the ξ–M plane for different Mach numbers where $\alpha = 0.5$ and $\beta = 0.272$.

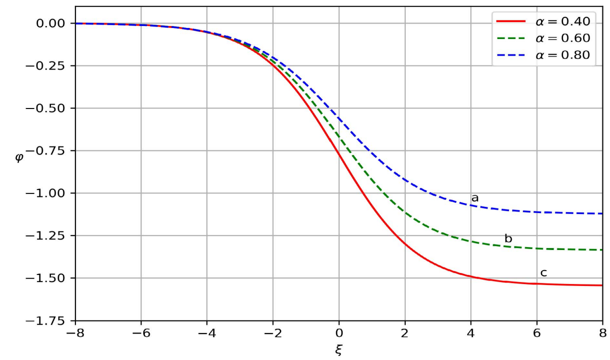

Fig. 10. DLs for different ratios of hot to cold electron equation density (α) are shown, where $\beta = 0.271$ and $M = 1.1$.

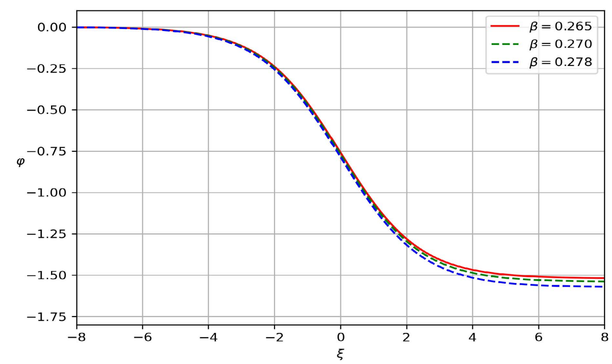

Fig. 11. DLs for different coefficients for the relativistic degeneracy pressure (β), where $\alpha = 0.4$ and $M = 1.1$.

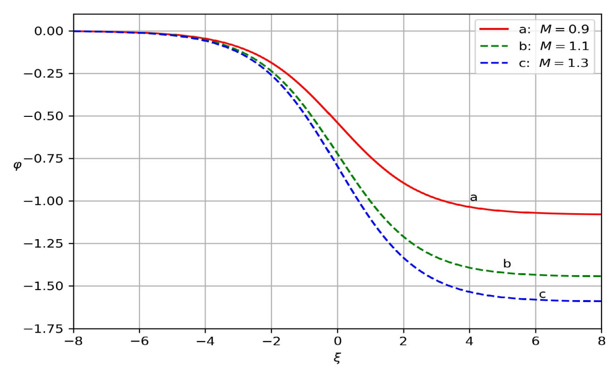

Fig. 12. DLs for different Mach numbers where $\alpha = 0.5$ and $\beta = 0.271$.

where $A_3 = (1/(54\beta^3) + (5\alpha^2)/(2M^6))$. From (17) we can obtain

$$\frac{1}{2}\left(\frac{d\varphi}{d\xi}\right)^2 = \frac{A_1}{2}\varphi^2 - \frac{A_2}{3}\varphi^3 + \frac{A_3}{4}\varphi^4. \tag{18}$$

The DLs are formed provided the BCs are $((d\varphi)/(d\xi))^2$ at $\varphi = 0$ and $\varphi = \varphi_m$, where $\varphi_m = (2A_2)/(3A_3)$. Therefore, the conditions for the existence of the DLs are

$$9A_1A_3 = 2A_2^2. \tag{19}$$

Using the tanh approach, we get the DL solution as shown below

$$\varphi(\xi) = \frac{\varphi_m}{2}\left[1 \mp \tanh\left(\frac{\xi}{\Delta}\right)\right] \tag{20}$$

where $\Delta = (3(4A_3)^{1/2})/A_2$. The parametric variations in DLs are shown in Figs. 10–12, where we can see how the potential changes with electron density, relativistic degeneracy pressure, and Mach number. The scenario where the hot to cold electron density (α) increases (Fig. 10) yields a qualitatively akin result: as the electrons drift further away from their equilibrium, the EA DL amplitude decreases, as shown by the formation in Fig. 10 with $M = 1.1$. This makes it possible to review EA DLs with small amplitudes with Mach numbers greater than 1. In Figs. 11 and 12, as the relativistic degeneracy pressure (β) and Mach number M increase, the negative EA DL compresses, enabling compressive EA DL to arise.

To procure small-amplitude stationary structure, we apply reductive perturbation technique in Section IV.

IV. Low-Frequency Acoustic Modes

Low-frequency acoustic waves are generated initially and propagate as a stationary structure defined by the KdV equation. We use stretched coordinates to construct the KdV equation for nonlinear EA wave propagation as follows [51]:

$$\zeta = \epsilon^{\frac{1}{2}}(x - V_p t) \tag{21}$$

$$\tau = \epsilon^{\frac{3}{2}} t. \tag{22}$$

Here, ϵ is the smallness parameter indicating weakening measurement of dispersion ($0 < \epsilon < 1$), and V_p is specified as EA wave phase speed (ω/k). We then expand n_c, n_h, u_c, u_h, and φ in power series of ϵ

$$\left.\begin{aligned} n_c &= 1 + \epsilon n_c^{(1)} + \epsilon^2 n_c^{(2)} + \epsilon^3 n_c^{(3)} + \ldots \\ n_h &= 1 + \epsilon n_h^{(1)} + \epsilon^2 n_h^{(2)} + \epsilon^3 n_h^{(3)} + \ldots \\ u_c &= 0 + \epsilon u_c^{(1)} + \epsilon^2 u_c^{(2)} + \epsilon^3 u_c^{(3)} + \ldots \\ u_h &= 0 + \epsilon u_h^{(1)} + \epsilon^2 u_h^{(2)} + \epsilon^3 u_h^{(3)} + \ldots \\ \varphi &= 0 + \epsilon \varphi^{(1)} + \epsilon^2 \varphi^{(2)} + \epsilon^3 \varphi^{(3)} + \ldots \\ \rho &= 0 + \epsilon \rho^{(1)} + \epsilon^2 \rho^{(2)} + \epsilon^3 \rho^{(3)} + \ldots \end{aligned}\right\}. \tag{23}$$

Now, we can express (1)–(5) concerning new variables ζ and τ, and substituting (23) in the resulting equation [(1)–(5) expressed with stretched coordinates]. We develop various sets of equations with varying the order of ϵ. The lowest power equations in ϵ yield

$$n_c^{(1)} = -\frac{\alpha}{{V_p}^2}\psi, \quad n_h^{(1)} = \frac{1}{\beta}\psi \tag{24}$$

$$u_c^{(1)} = -\frac{\alpha}{V_p}\psi, \quad u_h^{(1)} = \frac{V_p}{\beta}\psi \tag{25}$$

$$V_p = \sqrt{\beta} \tag{26}$$

where $\psi = \varphi^{(1)}$, where ψ is the measurement of potential, and linear dispersion relationship for the EA wave is represented by (26).

We construct a set of equations to the higher power of ϵ, and using these equations, we obtain

$$\frac{\partial \psi}{\partial \tau} + A\psi\frac{\partial \psi}{\partial \zeta} + B\frac{\partial^3 \psi}{\partial \zeta^3} = 0 \tag{27}$$

where

$$B = \frac{{V_p}^3}{2}, \quad A = -\frac{{V_p}^3}{2}\left[\frac{1}{\beta^2} + \frac{2\alpha}{{V_p}^4} + \frac{\alpha}{{V_p}^3}\right]. \tag{28}$$

Equation (27) is the well-known KdV equation, which defines the generation of nonlinear EA waves in the weakly degenerate ionospheric plasma.

To obtain the SW solution of the KdV (27), the independent variables are transformed into

$$\xi = \zeta - U_0 \tau \tag{29}$$

where U_0 denotes the SWs' speed. Implementing the BCs which are $\psi, (d\psi)/(d\xi), (d^2\psi)/(d\xi^2) \to 0$ at $\xi \to \pm\infty$, we get the possible SW solution of the KdV (27) equation [83] as

$$\psi = \psi_m \operatorname{sech}^2\left[\frac{\xi}{\Delta}\right]. \tag{30}$$

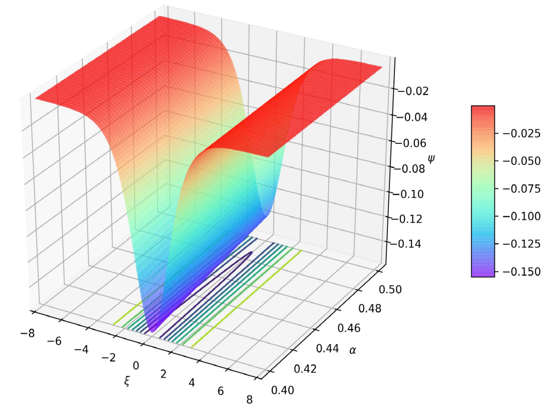

Fig. 13. Rarefactive KdV soliton's potential structure with α at $U_0 = 0.1$ and $\beta = 0.27$.

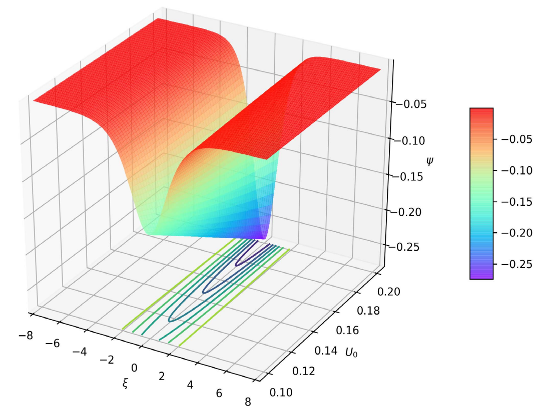

Fig. 14. Potential structure of KdV soliton with the variation in U_0 at $\alpha = 0.5$ and $\beta = 0.27$.

Here, the soliton amplitude is ψ_m. The soliton width is Δ. These are written as

$$\psi_m = \frac{3U_0}{A} \quad \text{and} \quad \Delta = \sqrt{\frac{4B}{U_0}}. \tag{31}$$

Now, we analyze the KdV SW structure [using (30)]. The coefficient of nonlinear effect plays a crucial role in determining compressive and rarefactive solitary potential profiles. Compressive SWs survive for $A > 0$, whereas rarefactive SWs exist for $A < 0$. Therefore, $A\ (\alpha = \alpha_c) = 0$, where the critical value of α is α_c, above (below) of which rarefactive (compressive) SW potential exists. But α_c is less than zero. As the ratio of hot and cold electron equation density is always a positive quantity, the only rarefactive potential SWs can exist.

For typical weakly relativistic ionospheric plasma parameters, Fig. 13 represents the variation in ψ versus ξ for $\alpha > \alpha_c$ to demonstrate the impact of the hot-to-cold electron density ratio on the soliton profile. The plot shows that as the electron density ratio increases, the amplitude of rarefactive solitary structures decreases, although the width decreases. It is worth noting that Bala *et al.* [84] observed a similar pattern of behavior of the solitary profiles. In Fig. 14, we have shown the profiles of KdV solutions with the variation in rarefactive potential along with U_0.

V. Solitary Structures at Critical Region

A. Derivation of mKdV Equation

Due to the vanishing of nonlinear terms at the critical regime, we proceed to obtain an mKdv equation. Thus, higher order calculation is essential to describe the nonlinear effect at the critical parameter. From second-order calculation of ϵ, we derive the KdV equation [27]. If we choose $\alpha = \alpha_c$ (which is a solution $A = 0$), the amplitude of SWs becomes infinite and fails to describe the nonlinear effect of perturbation. We have used a third order of ϵ to derive mKdV and introduce a new stretching variable [51] as follows:

$$\zeta = \epsilon(x - V_p t) \tag{32}$$

$$\tau = \epsilon^3 t. \tag{33}$$

Using (32) and (33) in (1)–(5) and (23), we calculate the expression for $n_c^{(1)}$, $n_h^{(1)}$, $u_c^{(1)}$, $u_h^{(1)}$, and V_p [which are the same as given in (24)–(26)]. For the next higher order of ϵ, we get

$$n_c^{(2)} = \frac{3\alpha^2}{2{V_p}^4}\psi^2 - \frac{\alpha}{{V_p}^2}\varphi^{(2)} \tag{34}$$

$$n_h^{(2)} = \frac{1}{\beta^2}\psi^2 + \frac{1}{\beta}\varphi^{(2)} \tag{35}$$

$$u_c^{(2)} = \frac{\alpha^2}{2{V_p}^3}\psi^2 - \frac{\alpha}{V}\varphi^{(2)} \tag{36}$$

$$u_h^{(2)} = \frac{V_p}{\beta}\varphi^{(2)} \tag{37}$$

$$\rho^{(2)} = \frac{1}{2}A\psi^2 = 0. \tag{38}$$

Another set of equations is obtained for the third-order approximation of ϵ. After combining them, we obtain the following equation:

$$\frac{\partial\psi}{\partial\tau} + D\psi^2\frac{\partial\psi}{\partial\zeta} + B\frac{\partial^3\psi}{\partial\zeta^3} = 0 \tag{39}$$

where $D = BQ$ and

$$B = \frac{V_p^3}{2} \tag{40}$$

$$Q = \frac{15\alpha^2}{2V_p^6} - \frac{1}{\beta^3}. \tag{41}$$

Equation (39) is the so-called mKdV equation. The transformation $\xi = \zeta - U_0\tau$ is used to obtain a stationary solution. U_0 is the velocity of the solitons. Therefore, the stationary localized solution is given by

$$\psi = \psi_m \,\mathrm{sech}\left[\frac{\xi}{\delta}\right]. \tag{42}$$

Here, ψ_m is the soliton amplitude. The width of the solitons is δ, which are

$$\psi_m = \sqrt{\frac{6U_0}{D}}, \quad \delta = \sqrt{\frac{B}{U_0}}. \tag{43}$$

For physically reasonable solutions, D must be positive (i.e., $D > 0$). So SWs do not exist for $-((2)/15)^{1/2} < \alpha_D < ((2)/15)^{1/2}$, where α_D is the solution for $D = 0$.

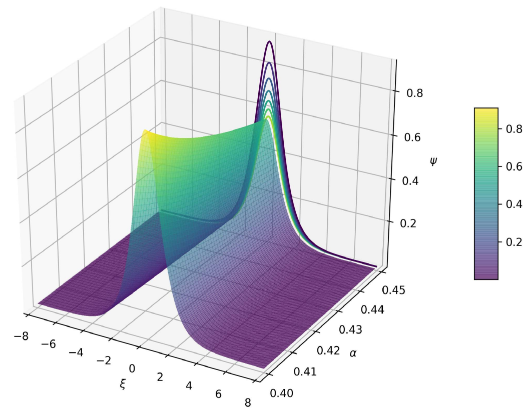

Fig. 15. Different SW structures of the mKdV equation with α at $U_0 = 0.1$ and $\beta = 0.27$.

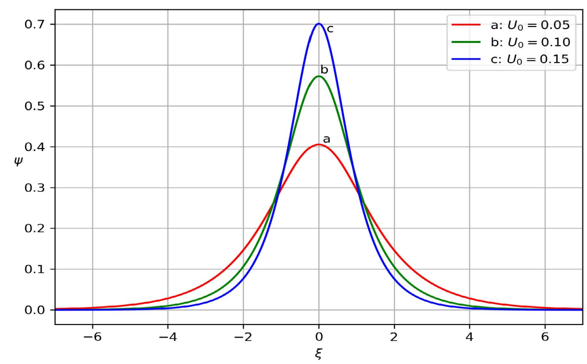

Fig. 16. SW structure of mKdV soliton for different U_0, where $\beta = 0.272$ and $\alpha = 0.45$.

For physically reasonable solution, $\alpha_D > 0$. Fig. 15 shows small-amplitude mKdV soliton with the variation in α where decreasing α increases the inertia and the amplitude of SWs is also increased. Fig. 16 shows the mKdV soliton for the variation in U_0. Though mKdV solitons have finite solution close to α_c, it has no DL solution. The amplitude of the EA SWs decreases with the increment of U_0 due to the increased inertia effects. Only positive potentials have been observed for the solitary profile of mKdV, as shown in this figure. We derived the SG equation, which yields the solutions for SW and DL as well.

B. Derivation of the SG Equation

It suffices to say that (38) corresponds to $A = 0$, but $\psi \neq 0$. SW does not exist at $A = 0$ since there is no nonlinear term. As we can see, A is a function of α and β. So, at a fixed value of β, there exist a threshold of α $(= \alpha_c)$, where such $A = 0$. For α near its critical value (α_c), $A = A_0$ can be written as

$$A_0 \simeq s\left(\frac{\partial A}{\partial\alpha}\right)_{\alpha=\alpha_c}|\alpha - \alpha_c| = s\epsilon A_\alpha \tag{44}$$

where $A_\alpha = -(1/2 + 1/V_p)$, and $|\alpha - \alpha_c|$ is a dimensionless and small parameter. This is used to specify the parameter of expansion ϵ as $|\alpha - \alpha_c| \simeq \epsilon$. s stands for the sign of $(\alpha - \alpha_c)$, i.e., for $s = 1$ for $\alpha > \alpha_c$ and for $s = -1$ for $\alpha < \alpha_c$. Thus, $\rho^{(2)}$ can be written as

$$\epsilon^2\rho^{(2)} \simeq \epsilon^3\frac{1}{2}s\psi^2 A_\alpha. \tag{45}$$

Therefore, this must be added in the Poisson equation of third order. We obtain a set of third equations after considering the next higher order of ϵ. The equations are

$$\frac{\partial n_c^{(1)}}{\partial \tau} - V_p \frac{\partial n_c^{(3)}}{\partial \zeta} + \frac{\partial F_c}{\partial \zeta} = 0 \quad (46)$$

$$\frac{\partial n_h^{(1)}}{\partial \tau} - V_p \frac{\partial n_h^{(3)}}{\partial \zeta} + \frac{\partial F_h}{\partial \zeta} = 0 \quad (47)$$

$$\frac{\partial u_c^{(1)}}{\partial \tau} - V_p \frac{\partial u_c^{(3)}}{\partial \tau} - \frac{\partial}{\partial \zeta}\left[\left(u_c^{(1)} u_c^{(2)}\right)\right] - \alpha \frac{\partial \varphi^{(3)}}{\partial \zeta} = 0 \quad (48)$$

$$\frac{\partial \varphi^{(3)}}{\partial \zeta} + n_h^{(1)} \frac{\partial \phi^{(2)}}{\partial \zeta} + n_h^{(2)} \frac{\partial \psi}{\partial \zeta} - \beta \frac{\partial n_h^{(3)}}{\partial \zeta} = 0 \quad (49)$$

$$\frac{\partial^2 \psi}{\partial \zeta^2} + \frac{1}{2} s A_\alpha \psi^2 - n_h^{(3)} - \frac{1}{\alpha} n_c^{(3)} = 0 \quad (50)$$

where $F_c = n_c^{(1)} u_c^{(2)} + n_c^{(2)} u_c^{(1)} + u_c^{(3)}$ and $F_h = n_h^{(1)} u_h^{(2)} + n_h^{(2)} u_h^{(1)} + u_h^{(3)}$. Now combining (34) to (38) and (46) to (50), the SG equation is obtained as follows:

$$\frac{\partial \psi}{\partial \tau} + P\psi \frac{\partial \psi}{\partial \zeta} + D\psi^2 \frac{\partial \psi}{\partial \zeta} + B \frac{\partial^3 \psi}{\partial \zeta^3} = 0 \quad (51)$$

where $P = sA_\alpha B$, and (51) is also called mixed modified KdV (mmKdV) equation because it includes both the terms ψ of KdV equation and ψ^2 of mKdV. SG equation is valid near critical value ($\alpha = \alpha_c$). If we put $sA_\alpha = A$ and $\alpha_D = ((2)/15)^{1/2}$, the SG equation reduces to KdV (27) which is derived using a lower order stretching [(21) and (22)] in Section IV.

C. SW Solution of the SG Equation

To evaluate SW solution of SG (51), we use the following transformation [85]:

$$\xi = \zeta - U_0 \tau. \quad (52)$$

Under steady-state condition, the transformation allows us to write

$$\frac{1}{2}\left(\frac{d\psi}{d\xi}\right)^2 + V(\psi) = 0 \quad (53)$$

where $V(\psi)$ is defined as the pseudopotential, expressed as

$$V(\psi) = -\frac{U_0}{2B}\psi^2 + \frac{sA_\alpha}{6}\psi^3 + \frac{Q}{12}\psi^4 \quad (54)$$

where U_0 and coefficient of dispersive effect (B) are always positive quantity. BCs for stationary wave solutions are

$$V(\psi)|_{\psi=0} = \frac{dV(\psi)}{d\psi}|_{\psi=0} = 0 \quad (55)$$

$$\frac{d^2 V(\psi)}{d\psi^2}|_{\psi=0} < 0 \quad (56)$$

$$V(\psi)|_{\psi=\psi_m} = 0. \quad (57)$$

The SW solution of (51) will exist when $V(\psi)|_{\psi=\psi_m} = 0$ condition is satisfied. The latter can be written as

$$U_0 = \frac{sA_\alpha B}{3}\psi_{m_\mp} + \frac{QB}{6}\psi_{m_\mp}^2 \quad (58)$$

$$\psi_{m_\mp} = \psi_m\left[1 \mp \sqrt{\frac{U_0}{V_0}}\right] \quad (59)$$

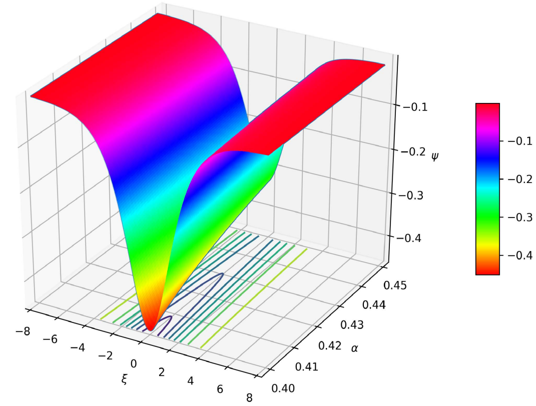

Fig. 17. Profile of rarefactive SGs varying with α for $U_0 = 0.05$, $\beta = 0.272$, and $s = 1$.

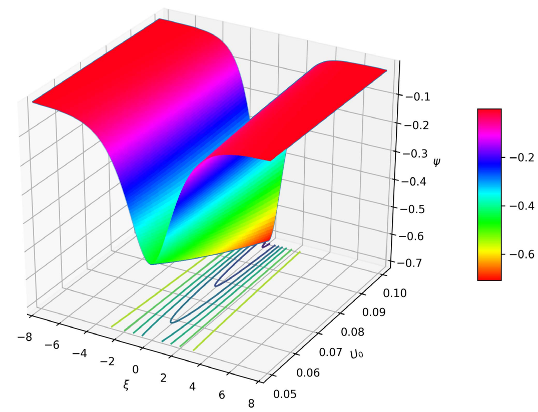

Fig. 18. Profile of rarefactive SGs varying with U_0 for $\alpha = 0.40$, $\beta = 0.272$, and $s = 1$.

where $\psi_m = -sA_\alpha/Q$ and $V_0 = (s^2 BA_\alpha^2)/(6Q)$. Now using (59) and (54) in (53), we get

$$\left(\frac{d\psi}{d\xi}\right)^2 + \frac{Q}{6}\psi^2\left(\psi - \psi_{m_-}\right)\left(\psi - \psi_{m_+}\right) = 0. \quad (60)$$

Therefore, the SW solution of (59) is expressed as

$$\psi = \left[\frac{1}{\psi_{m_+}} - \left(\frac{1}{\psi_{m_+}} - \frac{1}{\psi_{m_-}}\right)\cosh^2\left(\frac{\xi}{\delta}\right)\right]^{-1} \quad (61)$$

where the width of the soliton $\delta = (-(24)/(Q\psi_{m_+}\psi_{m_-}))^{1/2}$. Equation (61) represents the SW solution of (51). Therefore, it stands to reason that to get SWs, we should have $U_0 < V_0$; else, $\psi_{m_\mp}$ becomes imaginary. When $\alpha > \alpha_c$ (i.e., $s = 1$), (61) represents small-amplitude rarefactive soliton (Fig. 17). But $s = -1$, i.e., $\alpha < \alpha_c$, and (61) represents small-amplitude compressive soliton, which is not a physically valid solitary structure due to the ratio of hot and cold electron equation density. Fig. 18 denotes the variation in the potential structure of SGs with U_0.

D. Derivation of DLs Solution for SG Equation

To estimate the stationary DLs' solution of the SG equation, we consider a frame moving with speed U_0 and $\xi = \zeta - U_0\tau$ same as the implement in the SW structure solution. Now,

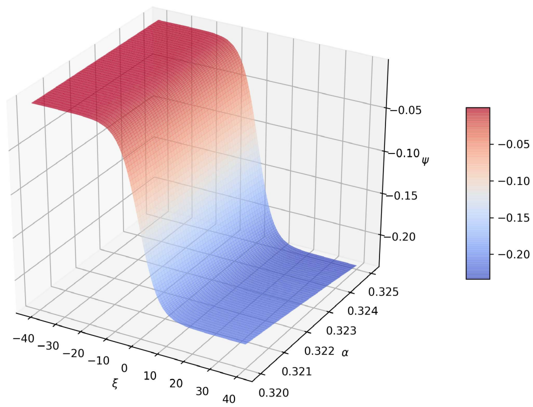

Fig. 19. DL structure is shown varying with α for $\beta = 0.272$ and $s = 1$.

impose all the necessary BCs required for the DL solution, including $\psi \to 0$, $(d\psi)/(d\xi) \to 0$, and $(d^2\psi)/(d\xi^2) \to 0$ at $\xi \to \infty$. Then satisfying condition (55) and (56) and implying them in (53), we find that the DL solution exists on condition

$$V(\psi)|_{\psi=\psi_m} = \frac{dV(\psi)}{d\psi}|_{\psi=\psi_m} = 0 \tag{62}$$

where the amplitude of the DL is given by

$$\psi_m = s\frac{6U_0}{A_\alpha B} \quad \text{and} \quad U_0 = -\frac{s^2 A_\alpha^2 B}{6Q}. \tag{63}$$

Putting ψ_m and (54) into (53), we have

$$\frac{d\psi}{\psi(\psi - \psi_m)} = -\sqrt{-\frac{Q}{6}}d\zeta. \tag{64}$$

Now, integrating (64) we obtain the stationary DL solution of SG equation which can be expressed as

$$\psi = \frac{\psi_m}{2}\left[1 \pm \tanh\left(\frac{\xi}{\Delta}\right)\right] \tag{65}$$

where the width of the DLs is given by

$$\Delta = \sqrt{\frac{24}{-\psi_m^2 Q}}. \tag{66}$$

It is clear from (63), (65), and (66) that DLs exist on condition Q < 0, i.e., $-(2/(15))^{1/2} < \alpha_D < (2/(15))^{1/2}$. Since $B > 0$ and $U_0 > 0$, (63), (65), and (66) indicate that the DLs have rarefactive potential when $s = 1$, i.e., $\alpha > \alpha_c$ and compressive potential when s $= -1$, i.e., $\alpha < \alpha_c$. Weakly relativistic degenerate plasma in a typical value range of β indicates $\alpha_D > \alpha_c$ which confirms that DLs exist only for rarefactive potential (shown in Fig. 19), a similar result to Fig. 10, as the electrons drift further away from their equilibrium.

VI. Rossby Solitons

The KdV equation, generally the contention between weak dispersion and weak nonlinearity, takes the lead for Rossby soliton, which was introduced by Long [86]. Afterward, with the advancement of SWs, the mK-dV condition is likewise produced to depict the development of Rossby soliton [87]. Rossby SWs that have been identified with KdV-type conditions are frequently called classical Rossby solitons. The extraordinary feature of this sort of SW is the waveform invariance and speed during interaction and propagation. Yet, when KdV-type solitons are applied to portray the genuine perceptions for SWs, there are two drawbacks: 1) the KdV soliton can push toward a specific direction. On the other hand, the real SWs can propagate in both the directions and 2) real observation leads to larger velocity for KdV soliton.

As u_{c0} is the basic flow, i.e., unperturbed velocity of the cold electron stream, then the dispersion relationship would be

$$\omega = -ku_{c0} \pm \sqrt{k^2u_{c0}^2 - \frac{\left(k^2u_{c0}^2 + \beta k^4 u_{c0}^2 - \beta k^2\right)}{\left(1+\beta k^2\right)}}. \tag{67}$$

To derive the long wave solution, let u be the wave disturbance of linear RWs given by

$$\left.\begin{aligned} u &= e^{i(kx-\omega t)} \\ \text{Therefore, } \frac{\partial u}{\partial t} &= -i\omega u;\ \frac{\partial u}{\partial x} = iku \end{aligned}\right\}. \tag{68}$$

Following the algebric steps suggested by Zhongming *et al.* [88], we get the nonlinear equation for long wave as

$$\frac{\partial u}{\partial t} - u\frac{\partial u}{\partial x} - R\frac{\partial u}{\partial x} - C\frac{\partial^3 u}{\partial x^3} + S\frac{\partial^5 u}{\partial x^5} = 0 \tag{69}$$

where

$$\left.\begin{aligned} R &= \left(\frac{5u_{c0}^4 + \beta^2 u_{c0}^4 + \beta^2 - 6\beta u_{c0}^2}{8u_{c0}^3}\right) \\ C &= \left(\frac{2\beta^3 u_{c0}^2 + \beta^3 - 4\beta^2 u_{c0}^2}{4u_{c0}^3}\right) \\ \text{And, } S &= \left(\frac{5\beta^2 u_{c0}^4 - 3\beta^4 u_{c0}^4 + 3\beta^4 - 2\beta^3 u_{c0}^2}{8u_{c0}^3}\right) \end{aligned}\right]. \tag{70}$$

It is quite impossible to obtain a precise analytic solution of (69). We will make some replacements to get a solution which is given by

$$\left.\begin{aligned} u &= v(\bar{\zeta});\ \bar{\zeta} = \sqrt{\lambda}(x_1 + qt_1) \\ \lambda &= -\frac{a}{3} - S\lambda_1 + S^2\lambda_2 \\ q &= \left(R - \frac{a}{3} - S\beta_1 + S^2\beta_2\right) \end{aligned}\right\} \tag{71}$$

where

$$\left.\begin{aligned} \lambda_1 &= \frac{5S^2}{36R^2C^2} \\ \lambda_2 &= \frac{5S^3}{108R^3C^3} \end{aligned}\right\}. \tag{72}$$

We get the solution of (69) as

$$u(x,t) = -U\,\mathrm{sech}^2\frac{\bar{\zeta}}{2}\left[1 - \frac{5}{4}S\tanh^2\frac{\bar{\zeta}}{2} - S^2\left(\frac{35}{16}\tanh^2\frac{\bar{\zeta}}{2} - \frac{155}{48}\tanh^4\frac{\bar{\zeta}}{2}\right)\right]. \tag{73}$$

Here, U is the amplitude. A mathematical estimation is been done using the physical values of parameters and an oscillatory wave, i.e., RW is acquired, along with its dispersion relation. Figs. 20 and 21 depict the existence of oscillatory Rossby SW with the variation in u_{c0} at $\beta = 0.271$.

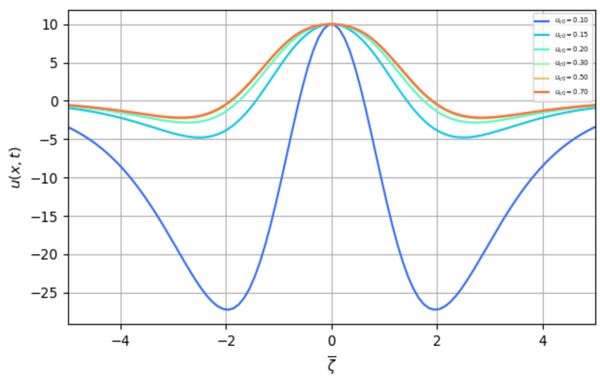

Fig. 20. Oscillatory Rossby SW for different u_{c0}, where $\beta = 0.271$.

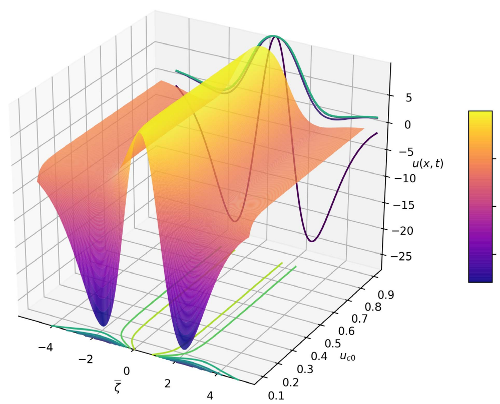

Fig. 21. 3-D oscillatory Rossby SW for varying u_{c0}, where $\beta = 0.271$.

VII. Results and Discussion

Apparently, it appears that for a wide range of parametric influences, solitons and DLs may be generated in this plasma, but a realistic range of values of the different parameters (α, β, M, or U_0) reveals solitary structures and DL is the only option. In our weakly relativistic dense plasma, we have ignored the quantum diffraction effect and the electron exchange interaction effect. This is justified since the quantum tunneling, and in the exchange, interaction corresponds to ultradense plasma (density $\sim 10^{32-34}/\mathrm{m}^3$). We have considered two kinds of electrons, viz. warm inertialess that provides the restoring force and the cold inertial one that provides the inertia. When both these categories of electrons are involved in the dynamic process, small-amplitude electrostatic oscillation begins to occur, which interacts to form nonlinear solitary structures. For this small-amplitude electric perturbation, the pseudopotential (13) is expanded up to the second order. An extension to the higher order would amount to the formation of DLs. With reference to Fig. 1 it is clearly said that as the value of α decreases, the maxima in the pseudopotential increases, suggesting that the decrease in the contribution of inertia. Higher value of α means that the hot electrons provide more oscillatory contribution such that solitary formation is more feasible. While hot electron density increases, mean electrons are more mobile so that double-layer formation is apparent. On the other hand, a slight increase in the value of β increased the maxima at the tail in the pseudopotential profile, much like the contribution of α. Here, β which signifies the equilibrium density of degenerate hot electron which is influential in providing mobility and higher restoring force Fig. 2. Finally, from Fig. 3 we can clearly observe that the slight change in wave frame velocity can enhance the chances of SW formation due to the coordinated correlation among the warm electrons.

From Figs. 4–6, we may conclude that equilibrium warm electron density causes the solitary profile to be shortened. This is interpreted as the negative effect of kinetic motion on nonlinear interaction. The higher value of the relativistic degeneracy factor (β) has a positive effect in enhancing the nonlinear effect. Wave frame velocity (M) has a similar effect in increasing the amplitude of soliton with no effect with the width (unlike Fig. 7, it is conclusive that higher potential corresponding to SW contour is SW contour obtained within a small range of hot-to-cold equilibrium density ratio (α). For its higher values, the solitons have a wider spread in the ξ–α plane. Relativistic degeneracy on the other hand a reverse effect as shown in 8. Further from Fig. 9, we observe a symmetric breakdown in the ξ–M plane, which might be due to the drifting of an SW. From Fig. 10, it is evident that the higher the density of degenerate electrons, the lower will be the potential corresponding to this attributed to the interplay of mobility and density of degenerate electron. Fig. 11 shows the opposite effect due to the relativistic degenerate pressure (represent β). However, it is to be noted that for the higher value of β implying high potential separation due to a balance between ion density and warm electron density. Fig. 12 shows effects opposite to that of α but nearly similar to β suggesting the fact that drift negatively influence the charge separation. It is clear from Fig. 13 that a small increase in warm electron equilibrium density decreases the amplitude of rarefactive soliton. But amplitude is increased with increasing wave frame velocity (Fig. 14). A negative effect has been noticed while cold electron provides less inertia to compressive mKdV soliton (Fig. 15). The width of mKdV soliton decreases, and amplitude increases with increasing wave frame velocity (Fig. 16). The magnitude of amplitude of rarefactive SGs has been decreased with decreasing the ratio of hot and cold electron equilibrium density above the critical regime, and a positive effect has been observed for increasing wave frame velocity (Figs, 17 and 18). In the case of DLs, as shown in Fig. 19, a small increase in equilibrium density will enhance the potential separation between two layers. For Rosbby SW, a minor change in wave frame velocity at low frame velocity region influences particle to oscillate as observed from Figs. 20 and 21.

The confined electron and ion populations must be sustained by 1) electrostatic potential; 2) collision, implying backscattering; and 3) magnetic mirrors for a DL to exist. The latter two processes are not possible in this problem since the DLs are electrically balanced and we must not leave out the electrostatic pressure in DL in which the ramp pressure of the free particles after acceleration or instantaneously earlier on retardation (if DL is drifting backward) will dominate [60]. We further checked the Bohm criteria $n_e u_{\mathrm{De0}}^2 > 2kT_{\mathrm{fe}}$ had obtained the minimum drift velocity of electron while entering the DL if $n_e u_{\mathrm{De0}}^2 = k(\gamma T_{\mathrm{fe}} + T_{\mathrm{ti}})$ where $u_{\mathrm{De}o}$ = Electron equilibrium drift velocity, γ = adiabatic constant, T_{fe} = free electron temperature, and T_{ti} = ion thermal temperature which is given by Maxwellian distribution implying a critical current density below which a DL cannot be sustained and is given by $J_e = ne(2kT_e/m_e)^{1/2}$.

VIII. Conclusion

Finally, we conclude that stationary structures are formed in a range of relativistic degeneracy pressure 0.265–0.278, and the ratio of hot and cold electron equilibrium density is between 0.3 and 0.8. Only rarefactive KdV SW will exist for typical plasma parameters, and compressive KdV soliton is not valid. MkdV compressive soliton will form for $\alpha > (2/(15))^{1/2}$. Whereas DLs will construct for $\alpha < (2/(15))^{1/2}$. Higher wave frame velocity forms large-amplitude SW, and at lower wave frame velocity, the only possibility is small-amplitude KdV solitary structures. RW was observed for higher order of perturbation. Around critical regime, compressive mKdV and rarefactive Gardner solitary structures are formed. In DLs, highly accelerating hot electrons enhance the separation between two layers. Rarefactive KdV soliton solutions in weakly relativistic plasma are also found by Mace *et al.* [89] which agree with our results. Lower hybrid solitary structures (LHSSs) were detected at altitudes ranging from 10 to 100 m in the auroral ionosphere on the upper side (Schuck *et al.* [90]). Concerning our findings and also as per earlier theoretical work carried out by Joyce and Hubbard [91] and others, we can conclude that for weak DLs, no such critical current density exists because an arbitrary number of free electrons can reverse their motion without disturbing the charge balance criteria. Here, the densities of the forward and backward moving electron species become comparable, and their free electron thermal energies become of the order of $e\varphi_0$ (PE of DL). On the contrary, two possibilities may occur if this particle stream leads to a higher current in plasma: 1) the current channel cross section will grow in plasma and 2) the DL will increase the potential drop. Our observation may help understand the planetary wave-type modulated electron density profiles observed by "Canadian Advanced Digital Ionosonde (CADI)" [92] in the equatorial ionospheric anomaly region [93].

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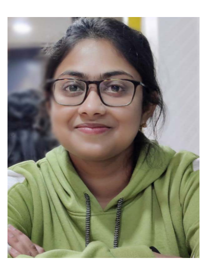

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